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$p - p'$ system with B field and projection operator noncommutative solitons

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Abstract: We study the system of the Dp'-brane with Dp-brane ($p < p'$) inside, in the case where B_{ij} field is a nonvanishing constant. In order to understand how the Dp-brane is viewed from the Dp'-brane worldvolume theory, we investigate the process in which the Dp-brane is probed with p'-p' open string. We calculate scattering amplitudes among p-p' open strings and p'-p' open strings and show that not only the Weyl transform of the projection operator onto the ground state but also those onto higher excited states emerge as multiplicative factors of the amplitudes.

Keywords: Bosonic Strings, Superstrings and Heterotic Strings, D-branes, Non-Commutative Geometry.

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Contents

1. Introduction

String theory with a constant NS-NS two-form B field background has several interesting features [1, 2, 3]. Among others, it has been found out that when D-branes accompany this system the worldvolume of the D-branes becomes noncommutative [4]–[8].

In this paper we study the p-p' system: the system of a Dp'-brane into which a Dp-brane ($p < p'$) is embedded, with a constant B field background. We would like to approach this system in terms of perturbative string theory. Several aspects of the p-p' system have been analyzed in the framework of string theory [9]–[12]. In the present paper, along the line of the analysis of [13], we investigate how the lower dimensional D-brane (i.e. Dp-brane) is viewed from the higher dimensional D-brane (i.e. Dp'-brane). For this purpose, we examine the process in which the Dp-brane is probed with p'-p' open strings. In the perturbative string theory, D-branes couple to the strings through the open strings attaching on them. Among the open strings ending on the Dp-brane, the p-p' open string directly couples to the p'-p' open string. Therefore the consideration of the process probing the Dp-brane with the p'-p' string leads us to evaluate scattering amplitudes among p-p' open strings and p'-p' open strings.

In refs. [13] and [14], from the evaluation of the N-point scattering amplitudes consisting of two ground state vertex operators of the p-p' open string and $(N-2)$ gauge field vertex operators of the p'-p' open string, it has been concluded that in the zero slope limit the form factor of the Dp-brane becomes precisely the Weyl transform of the projection operator $|0\rangle\langle 0|$, which is a classical solution obtained by [15] in the noncommutative field theory. This fact suggests that the Dp-brane within the noncommutative Dp'-brane should be described as the noncommutative soliton. Consistent results with this observation have been obtained from the analyses in terms of field theory [16]–[21]. Such coincidence leads us to the question whether it is possible to read from the string scattering amplitudes the projection operators $|\mathrm{m}\rangle\langle \mathrm{m}|$ $(\mathrm{m}=1,2,\ldots)$ besides that onto the harmonic oscillator ground state $|0\rangle\langle 0|$.

One of the remarkable properties of the p-p' system with B field is that there remains a large number of light states in an appropriate zero-slope limit in the spectrum of p-p' open string [8, 22]. These states are generated by multiplying the "almost-zero-modes". It is natural to expect that this tower of light states should play a role in the description of the Dp-brane within Dp'-brane as noncommutative solitons both in low energy field theory and in string theory. In order to understand the roles played by such a large number of light states, in this paper we compute three point scattering amplitudes which consist of one vertex operator of the p'-p' open string and two vertex operators of the p-p' open string corresponding to the states excited by some almost-zero-modes from the ground state. From the momentum dependent multiplicative factors in these amplitudes, we would like to read information as to the Dp-brane on the noncommutative Dp'-brane worldvolume.

This paper is organized as follows. In the next section, we briefly review the worldsheet properties of the p-p' open string. In section 3, we examine the vertex operators of the p-p' open string corresponding to the states excited by the almost-zero-modes and calculate the three point scattering amplitudes among two such vertex operators of the p-p' open string and one vertex operator of the p'-p' open string. In this section we concentrate on bosonic string theory. For the p'-p' string vertex operators, we prepare the tachyon vertex operator in section 3.2 and the noncommutative gauge field vertex operator in section 3.3. We find that Weyl transforms of projection operators emerge as multiplicative factors in the scattering amplitudes. In section 4, we extend the analysis into superstring theory. The final section is devoted to summary and discussions. In appendix A, we present formulae of the Weyl ordering prescription. In appendices B, C and D, some details of the calculation of the amplitudes are given.

2. Two Point Functions and Renormal Ordering

In order to fix the notation, in this section we review some basic properties of the p-p' system with a constant B field. In this paper we consider the situation where $p < p'$ and both of p and p' are even integers.

The setup of the system is the same as that in [13], i.e. a Dp-brane extends in the $x^0, x^1, \dots x^p$-directions and a Dp'-brane extends in the $x^0, x^1, \dots x^{p'}$-directions. The space-time is flat with the metric

$$g_{\hat{\mu}\hat{\nu}} = \left(\begin{array}{c|c|ccc} -1 & & & & \\ \hline & g_{kl} & & & \\ \hline & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{array}\right) , \qquad g_{kl} = \varepsilon \delta_{kl} \quad (k, l = 1, \dots, p') . \qquad (2.1)$$

We bring $B_{\hat{\mu}\hat{\nu}}$ into a canonical form

$$B_{\hat{\mu}\hat{\nu}} = \frac{\varepsilon}{2\pi\alpha'} \left(\begin{array}{c|ccccc|ccc} 0 & & & & & & & & \\ \hline & 0 & b_1 & & & & & & \\ & -b_1 & 0 & & & & & & \\ & & & \ddots & & & & & \\ & & & & 0 & b_{p'/2} & & & \\ & & & & -b_{p'/2} & 0 & & & \\ \hline & & & & & & 0 & & \\ & & & & & & & \ddots & \\ & & & & & & & & 0 \end{array}\right) . \qquad (2.2)$$

In what follows, we will write the string supercoordinates along the Dp'-brane as $\mathbb{X}^M(\mathbf{z}, \overline{\mathbf{z}}) = (\mathbb{X}^\mu(\mathbf{z}, \overline{\mathbf{z}}), \mathbb{X}^m(\mathbf{z}, \overline{\mathbf{z}}))$ $(M = 0, 1, \dots, p')$, where $\mu(= 0, 1, \dots, p)$ and $m(= p+1, \dots, p')$ denote the directions parallel and perpendicular to the Dp-brane respectively. In terms of component fields the superfields $\mathbb{X}^M(\mathbf{z}, \overline{\mathbf{z}})$ are expressed as

$$\mathbb{X}^M(\mathbf{z}, \overline{\mathbf{z}}) = \sqrt{\frac{2}{\alpha'}} X^M(z, \overline{z}) + i\theta\psi^M(z) + i\overline{\theta}\widetilde{\psi}^M(\overline{z}) . \qquad (2.3)$$

The string coordinates $\mathbb{X}^M(\mathbf{z}, \overline{\mathbf{z}})$ of the p'-p' open string and $\mathbb{X}^\mu(\mathbf{z}, \overline{\mathbf{z}})$ of the p-p' open string obey the mixed boundary conditions at the both ends. These coordinates

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are expanded in an integral power series of z and $\overline{z}$ [6, 7, 13]. The two-point functions of these string coordinates are given in [1, 2, 3, 11, 8]. When computed on the negative real axis, this becomes [8, 13]

$$\begin{aligned}\mathbb{G}^{\mu\nu}(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2) &\equiv \mathbb{G}^{\mu\nu}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\Big|_{\substack{z_a=e^{\tau_a+i\pi}\\ \theta_a=\overline{\theta}_a}} \\ &\equiv \langle 0|\mathcal{R}\mathbb{X}^{\mu}(\mathbf{z}_1,\overline{\mathbf{z}}_1)\mathbb{X}^{\nu}(\mathbf{z}_2,\overline{\mathbf{z}}_2)|0\rangle\Big|_{\substack{z_a=e^{\tau_a+i\pi}\\ \theta_a=\overline{\theta}_a}} \\ &= -2G^{\mu\nu}\ln(e^{\tau^{(1)}}-e^{\tau^{(2)}}+\theta^{(1)}\theta^{(2)})^2-\frac{i}{\alpha'}\theta^{\mu\nu}\epsilon(\tau^{(1)}-\tau^{(2)})\,,\end{aligned} \tag{2.4}$$

where $\mathcal{R}$ stands for the radial ordering, $G^{\mu\nu}$ and $\theta^{\mu\nu}$ are the inverse of the open string metric $G_{\mu\nu}$ and the noncommutativity parameter defined in [8] respectively, and $\epsilon(x)$ denotes the sign function.

The string coordinates $\mathbb{X}^m(\mathbf{z},\overline{\mathbf{z}})$ of p-p' open string obey the Dirichlet boundary condition at the $\sigma=0$ end attaching on the Dp-brane and obey the mixed boundary condition at the $\sigma=\pi$ end attaching on the Dp'-brane. These coordinates are expanded in a non-integral power series of z and $\overline{z}$ [8, 13]. In the following we will concentrate on the case where $p'=p+2$. Since we have brought the background B_{MN} into the canonical form (2.2), we can readily generalize the following analysis into more generic p' cases. We complexify $\mathbb{X}^m(\mathbf{z},\overline{\mathbf{z}})=(\mathbb{X}^{p+1}(\mathbf{z},\overline{\mathbf{z}}),\mathbb{X}^{p+2}(\mathbf{z},\overline{\mathbf{z}}))$ as

$$\begin{aligned}\mathbb{Z}(\mathbf{z},\overline{\mathbf{z}}) &\equiv \mathbb{X}^{p+1}(\mathbf{z},\overline{\mathbf{z}})+i\mathbb{X}^{p+2}(\mathbf{z},\overline{\mathbf{z}})\equiv\sqrt{\frac{2}{\alpha'}}Z(z,\overline{z})+i\theta\Psi(z)+i\overline{\theta}\widetilde{\Psi}(\overline{z})\,,\\ \overline{\mathbb{Z}}(\mathbf{z},\overline{\mathbf{z}}) &\equiv \mathbb{X}^{p+1}(\mathbf{z},\overline{\mathbf{z}})-i\mathbb{X}^{p+2}(\mathbf{z},\overline{\mathbf{z}})\equiv\sqrt{\frac{2}{\alpha'}}\overline{Z}(z,\overline{z})+i\theta\overline{\Psi}(z)+i\overline{\theta}\widetilde{\overline{\Psi}}(\overline{z})\,.\end{aligned} \tag{2.5}$$

The mode expansions of these complex fields are

$$\begin{aligned}Z(z,\overline{z}) &= i\sqrt{\frac{\alpha'}{2}}\sum_{n\in\mathbb{Z}}\frac{\alpha_{n-\nu}}{n-\nu}\left(z^{-(n-\nu)}-\overline{z}^{-(n-\nu)}\right)\,,\\ \overline{Z}(z,\overline{z}) &= i\sqrt{\frac{\alpha'}{2}}\sum_{m\in\mathbb{Z}}\frac{\overline{\alpha}_{m+\nu}}{m+\nu}\left(z^{-(m+\nu)}-\overline{z}^{-(m+\nu)}\right)\,,\\ \Psi(z) &= \sum_{r\in\mathbb{Z}+1/2}b_{r-\nu}z^{-(r-\nu)-\frac{1}{2}}\,,\quad \widetilde{\Psi}(\overline{z})=-\sum_{r\in\mathbb{Z}+1/2}b_{r-\nu}\overline{z}^{-(r-\nu)-\frac{1}{2}}\,,\\ \overline{\Psi}(z) &= \sum_{s\in\mathbb{Z}+1/2}\overline{b}_{s+\nu}z^{-(s+\nu)-\frac{1}{2}}\,,\quad \widetilde{\overline{\Psi}}(\overline{z})=-\sum_{s\in\mathbb{Z}+1/2}\overline{b}_{s+\nu}\overline{z}^{-(s+\nu)-\frac{1}{2}}\,,\end{aligned} \tag{2.6}$$

where ν is defined [8] by

$$e^{2\pi i\nu}=-\frac{1+ib_{(p+2)/2}}{1-ib_{(p+2)/2}}\,,\quad 0<\nu<1\,. \tag{2.7}$$

The modes satisfy the commutation relations

$$[\alpha_{n-\nu}, \overline{\alpha}_{m+\nu}] = \frac{2}{\varepsilon}(n-\nu)\delta_{n+m}\,, \quad \{b_{r-\nu}, \overline{b}_{s+\nu}\} = \frac{2}{\varepsilon}\delta_{r+s}\,. \tag{2.8}$$

The fact that the string coordinates $Z(z,\overline{z})$ and $\overline{Z}(z,\overline{z})$ are expanded by a non-integral power series of z and $\overline{z}$ implies that a twist field $\sigma^+(z,\overline{z})$ and an anti-twist field $\sigma^-(z,\overline{z})$, both of which are mutually non-local with respect to Z and $\overline{Z}$, locate at the origin and at the infinity on the complex plane respectively and generate a branch-cut between themselves. For the fermionic coordinates the situation is the same. The mutually non-local fields in the fermionic sector are a spin field $s^+(z,\overline{z})$ and an anti-spin field $s^-(z,\overline{z})$. The (anti-) twist and the (anti-) spin fields serve as boundary condition changing operators [9, 10]. The fields σ^+ and s^+ create the incoming oscillator vacuum $|\sigma, s\rangle$ and the fields σ^- and s^- create the outgoing vacuum $\langle\sigma, s|$ [13]. The two-point function of $\mathbb{Z}$ and $\overline{\mathbb{Z}}$ evaluated on these vacua are obtained in [13]. On the negative real axis, i.e. on the D$(p+2)$-brane worldvolume, it takes the form [13] of

$$\begin{aligned}
&\boldsymbol{\mathcal{G}}^{Z\overline{Z}}(-e^{\tau_1},\theta_1|-e^{\tau_2},\theta_2) \equiv \\
&\equiv \boldsymbol{\mathcal{G}}^{Z\overline{Z}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\Big|_{\substack{z_a=e^{\tau_a+i\pi}\\ \theta_a=\overline{\theta}_a}} \equiv \langle\sigma,s|\mathcal{R}\mathbb{Z}(\mathbf{z}_1,\overline{\mathbf{z}}_1)\overline{\mathbb{Z}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)|\sigma,s\rangle\Big|_{\substack{z_a=e^{\tau_a+i\pi}\\ \theta_a=\overline{\theta}_a}} \\
&= \frac{8}{\varepsilon(1+b^2)}\left[\Theta(\tau_1-\tau_2)\mathcal{F}\left(1-\nu;\frac{e^{\tau_2}-\theta_1\theta_2}{e^{\tau_1}}\right)+\Theta(\tau_2-\tau_1)\mathcal{F}\left(\nu;\frac{e^{\tau_1}}{e^{\tau_2}-\theta_1\theta_2}\right)\right]\,,
\end{aligned} \tag{2.9}$$

where $\Theta(x)$ is the step function and $\mathcal{F}(\nu;z)$ is defined by using the hypergeometric function $F(a,b;c;z)$ as

$$\mathcal{F}(\nu;z) = \frac{z^\nu}{\nu}F(1,\nu;1+\nu;z) = \sum_{n=0}^{\infty}\frac{1}{n+\nu}z^{n+\nu}\,. \tag{2.10}$$

In this way, this system has two types of vacuum for the string coordinates $\mathbb{Z}$ and $\overline{\mathbb{Z}}$; the one is the SL$(2,\mathbb{R})$ invariant vacuum $|0\rangle$ and the other is the oscillator vacuum $|\sigma,s\rangle$. This implies that we can define two types of normal ordering associated with the vacua $|0\rangle$ and $|\sigma,s\rangle$ respectively. We denote the normal ordering associated with $|0\rangle$ by $:\;:$ and that associated with $|\sigma,s\rangle$ by $\circ\!\!\!\circ\;\circ\!\!\!\circ$. The relation between these normal orderings is described by the renormal ordering formula [13]:

$$\begin{aligned}
:\boldsymbol{\mathcal{O}}: &= \exp\left(\int d^2\mathbf{z}_1 d^2\mathbf{z}_2\;\boldsymbol{\mathcal{G}}^{Z\overline{Z}}_{\rm sub}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\frac{\delta}{\delta\mathbb{Z}(\mathbf{z}_1,\overline{\mathbf{z}}_1)}\frac{\delta}{\delta\overline{\mathbb{Z}}(\mathbf{z}_2,\overline{\mathbf{z}}_2)}\right)\circ\!\!\!\circ\,\boldsymbol{\mathcal{O}}\,\circ\!\!\!\circ\,, \\
\boldsymbol{\mathcal{G}}^{Z\overline{Z}}_{\rm sub}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) &\equiv \boldsymbol{\mathcal{G}}^{Z\overline{Z}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) - \mathbb{G}^{Z\overline{Z}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\,,
\end{aligned} \tag{2.11}$$

where $\boldsymbol{\mathcal{O}}$ is an arbitrary functional of $\mathbb{Z}$ and $\overline{\mathbb{Z}}$ and $\mathbb{G}^{Z\overline{Z}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)$ is complexification of $\mathbb{G}^{mn}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)$:

$$\begin{aligned}
\mathbb{G}^{Z\overline{Z}}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) &= \mathbb{G}^{p+1,p+1}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) + \mathbb{G}^{p+2,p+2}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) - \\
&\quad - i\mathbb{G}^{p+1,p+2}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2) - i\mathbb{G}^{p+2,p+1}(\mathbf{z}_1,\overline{\mathbf{z}}_1|\mathbf{z}_2,\overline{\mathbf{z}}_2)\,.
\end{aligned} \tag{2.12}$$

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We will take the zero-slope limit proposed by Seiberg and Witten [8] in which α' is sent to zero with the open string metric G_{MN} and the noncommutativity parameter θ^{MN} kept finite. This implies that

$$\begin{cases} \alpha' \sim \varepsilon^{1/2} \to 0 \\ g_{MN} \sim \varepsilon \to 0 \\ |b_I| \sim \varepsilon^{-1/2} \to \infty \quad (I = 1, \ldots, (p+2)/2) \end{cases} . \tag{2.13}$$

In this limit, whether b_I goes to $+\infty$ or $-\infty$ just depends on the convention. In what follows, we will send $b_{(p+2)/2}$ to $+\infty$ in the zero-slope limit. Thus in the zero-slope limit we have

$$\begin{aligned} \theta^{p+1\,p+2} &\equiv -\frac{2\pi\alpha' b_{(p+2)/2}}{\varepsilon\left(1 + \left(b_{(p+2)/2}\right)^2\right)} < 0\,, \\ 1 - \nu &\simeq \frac{1}{\pi b_{(p+2)/2}} \sim \varepsilon^{1/2} \to 0\,. \end{aligned} \tag{2.14}$$

3. Three point amplitudes and multiplicative factors in bosonic string theory

We will calculate three point amplitudes which consist of two vertex operators of the p-$(p+2)$ open string and one vertex operator of the $(p+2)$-$(p+2)$ open string. As we mentioned in the introduction, in this section we restrict ourselves to bosonic string theory. The computations in this section will help us to extend these investigations into the superstring case in the next section.

The open string vertex operators should be inserted at the boundary of the worldsheet, namely the real axis of the complex upper half plane (z-plane). We insert at $\xi \equiv \mathrm{Re}(z) = \xi^{(1)}$ and at $\xi = \xi^{(3)} (> \xi^{(1)})$ the vertex operators of the p-$(p+2)$ open string which contain the anti-twist field σ^- and twist field σ^+ respectively. Since the (anti-) twist field serves as the boundary condition changing operator, the interval of the worldsheet boundary between them, $\xi \in [\xi^{(1)}, \xi^{(3)}]$, is interpreted to be on the D$(p+2)$-brane and the remaining part be on the Dp-brane [9, 10]. Thus the location $\xi^{(2)}$ of the vertex operator of the $(p+2)$-$(p+2)$ open string should be $\xi^{(2)} \in [\xi^{(1)}, \xi^{(3)}]$.

3.1 Vertex operators of p-$(p+2)$ open string

In the zero-slope limit (2.13), there exist *almost-zero-modes* $\overline{\alpha}_{-1+\nu}$ and $\alpha_{1-\nu}$ in the bosonic sector of the p-$(p+2)$ open string. In the superstring case, these modes generate a tower of light states which survives the zero slope limit [8, 22]. This implies that in the low energy physics Dp-brane is perturbatively seen as a collection of such a large number of light states. This fact gives the possibility that we grasp some nontrivial D-brane physics in the perturbative analysis even after taking the zero-slope limit.

We consider the vertex operators $\mathcal{V}^{(+)}_{\rm n}(\xi;k_\mu)$ and $\mathcal{V}^{(-)}_{\rm m}(\xi;k_\mu)$ of the p-$(p+2)$ open string which correspond to the states excited by almost-zero-modes from the oscillator vacuum $|\sigma\rangle$:

$$\begin{aligned}\mathcal{V}^{(+)}_{\rm n}(\xi;k_\mu) &=: \exp\left(ik_\mu X^\mu\right)(\xi) : V^{(+)}_{\rm n}(\xi)\ ,\\ \mathcal{V}^{(-)}_{\rm m}(\xi;k_\mu) &=: \exp\left(ik_\mu X^\mu\right)(\xi) : V^{(-)}_{\rm m}(\xi)\ ,\end{aligned} \tag{3.1}$$

where $V^{(+)}_{\rm n}(\xi)$ and $V^{(-)}_{\rm m}(\xi)$ are the contributions from the string coordinates Z and $\overline{Z}$ defined by

$$\begin{aligned}&\lim_{\xi\to 0} V^{(+)}_{\rm n}(\xi)|0\rangle = (\overline{\alpha}_{-1+\nu})^{\rm n}|\sigma\rangle \qquad ({\rm n}=0,1,2\ldots)\ ,\\ &\lim_{\xi\to\infty} \xi^{2h}\langle 0|V^{(-)}_{\rm m}(\xi) = \langle\sigma|(\alpha_{1-\nu})^{\rm m} \qquad ({\rm m}=0,1,2,\ldots)\,.\end{aligned} \tag{3.2}$$

Here h is the conformal weight of the operator $V^{(-)}_{\rm m}(\xi)$,

$$h = \left({\rm m}+\frac{\nu}{2}\right)(1-\nu)\,. \tag{3.3}$$

We note that the momentum k_μ carried by $\mathcal{V}^{(\pm)}_{\rm m}(\xi;k_\mu)$ is a $(p+1)$ vector along the Dp-brane worldvolume. Taking the relation $(\alpha_{1-\nu})^\dagger = \overline{\alpha}_{-1+\nu}$ into account, one finds that the definition (3.2) means that the space-time field $\Phi^{(-)}_{\rm m}$ corresponding to the vertex operator $\mathcal{V}^{(-)}_{\rm m}(\xi;k_\mu)$ is hermitian conjugate to the field $\Phi_{\rm m}$ corresponding to $\mathcal{V}^{(+)}_{\rm m}(\xi;k_\mu)$: $\Phi^{(-)}_{\rm m} = \Phi^\dagger_{\rm m}$. The physical state conditions for $\mathcal{V}^{(+)}_{\rm m}(\xi;k_\mu)$ and $\mathcal{V}^{(-)}_{\rm m}(\xi;k_\mu)$ require the on-shell condition $\alpha' m^2 \equiv -\alpha' G^{\mu\nu}k_\mu k_\nu = ({\rm m}+\frac{\nu}{2})(1-\nu)-1$ for the field $\Phi^{(\dagger)}_{\rm m}$.

3.2 Probe with tachyon field

In this subsection we examine the process probing the Dp-brane with the tachyon field excited from the $(p+2)$-$(p+2)$ open string. The vertex operator of the tachyon is

$$\mathcal{V}_\phi(\xi;k_M) =: \exp\left(i\sum_{M=0}^{p+2} k_M X^M\right)(\xi) :=: \exp\left(i\sum_{\mu=0}^{p} k_\mu X^\mu\right)(\xi) : V_\phi(\xi;k_m)\,, \tag{3.4}$$

where $V_\phi(\xi;k_m)$ is the contribution from the fields Z and $\overline{Z}$:

$$\begin{aligned}&V_\phi(\xi;k_m) =: \exp\left[i\left(\kappa Z+\overline{\kappa}\overline{Z}\right)\right](\xi) :\ ,\\ &\quad\text{with}\quad \kappa \equiv \frac{1}{2}\left(k_{p+1}-ik_{p+2}\right)\ ,\quad \overline{\kappa} \equiv \frac{1}{2}\left(k_{p+1}+ik_{p+2}\right)\ .\end{aligned} \tag{3.5}$$

Note that the momenta carried by the vertex operators of the $(p+2)$–$(p+2)$ open string are $(p+2)+1$ dimensional vectors in the space-time while those carried by the p-$(p+2)$ open string are $p+1$ dimensional ones. The physical state conditions for the vertex operator $\mathcal{V}_\phi(\xi;k_M)$ require that the on-shell condition $\alpha' m^2 \equiv -\alpha' k_M k_N G^{MN} = -1$ should hold.

We would like to calculate the three point disc amplitude

$$\mathcal{A}_3^{\text{tachyon}} = c\int \frac{d\xi^{(1)}d\xi^{(2)}d\xi^{(3)}}{d^3F(\xi^{(1)},\xi^{(2)},\xi^{(3)})}\times \\ \times\langle 0|\,\mathcal{V}_{\mathrm{m}}^{(-)}\left(\xi^{(1)};k_\mu^{(1)}\right)\,\mathcal{V}_\phi\left(\xi^{(2)};k_M^{(2)}\right)\,\mathcal{V}_{\mathrm{n}}^{(+)}\left(\xi^{(3)};k_\mu^{(3)}\right)\,|0\rangle\,, \tag{3.6}$$

where c is an overall constant which we are not interested in, and $d^3F(\xi^{(1)},\xi^{(2)},\xi^{(3)})$ denotes the volume element of the isometry $\mathrm{SL}(2,\mathbb{R})$ gauge group generated by the conformal Killing vectors on the worldsheet:

$$d^3F(\xi^{(1)},\xi^{(2)},\xi^{(3)}) = \frac{d\xi^{(1)}d\xi^{(2)}d\xi^{(3)}}{(\xi^{(1)}-\xi^{(2)})(\xi^{(2)}-\xi^{(3)})(\xi^{(3)}-\xi^{(1)})}\,. \tag{3.7}$$

This amplitude measures the process where the field Φ_{n} propagating along the Dp-brane with momentum $k_\mu^{(3)}$ prepared at the initial state is scattered by the tachyon on the D$(p+2)$-brane and then at the final state the field Φ_{m} is observed moving along the Dp-brane with momentum $-k_\mu^{(2)}$.

Through the computations presented in appendix B, we obtain

$$\mathcal{A}_3^{\text{tachyon}} = c\prod_{\mu=0}^{p}\delta(k_\mu^{(1)}+k_\mu^{(2)}+k_\mu^{(3)})\exp\left[\sum_{1\le c<d\le 3}\frac{i}{2}\theta^{\mu\nu}k_\mu^{(c)}k_\nu^{(d)}\epsilon(\tau^{(c)}-\tau^{(d)})\right]\times \\ \times\sqrt{\mathrm{m}!\,\mathrm{n}!}\left(\frac{2(1-\nu)}{\varepsilon}\right)^{\frac{\mathrm{m}+\mathrm{n}}{2}}\mathcal{I}_{\mathrm{nm}}\left(b_{(p+2)/2},k_m^{(2)}\right), \tag{3.8}$$

where the momentum dependent factor $\mathcal{I}_{\mathrm{nm}}(b_{(p+2)/2},k_m^{(2)})$ is defined as

$$\mathcal{I}_{\mathrm{nm}} \equiv \begin{cases} \sqrt{\frac{\mathrm{n}!}{\mathrm{m}!}}\left(i\overline{\kappa}^{(2)}\sqrt{2\Xi}\right)^{\mathrm{m}-\mathrm{n}}\exp\mathcal{C}(\nu;k_m^{(2)})L_{\mathrm{n}}^{(\mathrm{m}-\mathrm{n})}(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}) \ (\mathrm{m}\ge\mathrm{n}) \\ \sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}}\left(i\kappa^{(2)}\sqrt{2\Xi}\right)^{\mathrm{n}-\mathrm{m}}\exp\mathcal{C}(\nu;k_m^{(2)})L_{\mathrm{m}}^{(\mathrm{n}-\mathrm{m})}(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}) \ (\mathrm{m}<\mathrm{n}) \end{cases}, \tag{3.9}$$

with

$$\Xi \equiv \frac{2\alpha'}{\varepsilon(1-\nu)\left(1+(b_{(p+2)/2})^2\right)}. \tag{3.10}$$

In eq. (3.9), $L_{\mathrm{n}}^{(\alpha)}(x)$ denotes the Laguerre polynomial

$$L_{\mathrm{n}}^{(\alpha)}(x) = \frac{x^{-\alpha}e^x}{\mathrm{n}!}\frac{d^{\mathrm{n}}}{dx^{\mathrm{n}}}\left(e^{-x}x^{\mathrm{n}+\alpha}\right) = \sum_{l=0}^{\mathrm{n}}(-1)^l\binom{\mathrm{n}+\alpha}{\mathrm{n}-l}\frac{x^l}{l!}\,, \tag{3.11}$$

$\mathcal{C}(\nu;k_m)$ is defined as

$$\mathcal{C}(\nu;k_m) = 2\alpha'\left\{\gamma+\frac{1}{2}\left(\psi(\nu)+\psi(1-\nu)\right)\right\}G^{Z\overline{Z}}\kappa\overline{\kappa}\,, \tag{3.12}$$

$$\gamma:\ \text{Euler constant}\,,\quad \psi(\nu):\ \text{digamma function}\,,$$

and $G^{Z\overline{Z}}$ is the open string metric in the complex notation in the x^{p+1}, x^{p+2} directions:

$$G^{Z\overline{Z}} = G^{p+1\,p+1} + G^{p+2\,p+2} - 2iG^{p+1\,p+2} = \frac{2}{\varepsilon\left(1+\left(b_{(p+2)/2}\right)^2\right)}. \tag{3.13}$$

Here we make some comments on the momentum dependent multiplicative factor $\mathcal{I}_{\mathrm{nm}}$. The gaussian damping factor $\exp\mathcal{C}(\nu; k_m^{(2)})(=\mathcal{I}_{00})$ included in all of the multiplicative factors $\mathcal{I}_{\mathrm{nm}}$ comes from the scattering process in which the almost-zero modes do not participate. This factor takes the same form as what is obtained in the superstring case [13]. The every other piece in $\mathcal{I}_{\mathrm{nm}}$ is the contribution from the almost-zero-modes. This originates from eq. (B.8).

Let us take the zero-slope limit (2.13). In this limit, $\mathcal{C}(\nu; k_m^{(2)}) \to -|\theta^{p+1\,p+2}|\kappa^{(2)}\overline{\kappa}^{(2)}$ [13, 14] and $\Xi \to |\theta^{p+1\,p+2}|$. This tells us that

$$\mathcal{I}_{\mathrm{nm}} \to \begin{cases} \sqrt{\frac{\mathrm{n}!}{\mathrm{m}!}}\left(i\sqrt{2|\theta^{p+1\,p+2}|}\overline{\kappa}^{(2)}\right)^{\mathrm{m-n}} e^{-|\theta^{p+1\,p+2}|\kappa^{(2)}\overline{\kappa}^{(2)}} L_{\mathrm{n}}^{(\mathrm{m-n})}\left(2|\theta^{p+1\,p+2}|\kappa^{(2)}\overline{\kappa}^{(2)}\right) & (\mathrm{m}\geq\mathrm{n}) \\ \sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}}\left(i\sqrt{2|\theta^{p+1\,p+2}|}\kappa^{(2)}\right)^{\mathrm{n-m}} e^{-|\theta^{p+1\,p+2}|\kappa^{(2)}\overline{\kappa}^{(2)}} L_{\mathrm{m}}^{(\mathrm{n-m})}\left(2|\theta^{p+1\,p+2}|\kappa^{(2)}\overline{\kappa}^{(2)}\right) & (\mathrm{m}<\mathrm{n}) \end{cases}. \tag{3.14}$$

This is precisely the Weyl transform $\widetilde{f}_{\mathrm{nm}}(k_m)$ of the operator $|\mathrm{n}\rangle\langle\mathrm{m}|$ on the noncommutative $\mathbb{R}^2$ space obtained in eq. (A.13). Consequently the momentum dependent multiplicative factor $\mathcal{I}_{\mathrm{nm}}$ which we read off from the amplitude for the process probing the Dp-brane with the tachyon field becomes the Weyl transform of the operator $|\mathrm{n}\rangle\langle\mathrm{m}|$ in the zero-slope limit. In the case of $\mathrm{m}=\mathrm{n}$ in particular, this becomes the projection operator $|\mathrm{m}\rangle\langle\mathrm{m}|$. This is what we wanted.

3.3 Probe with gauge field

In this subsection we would like to probe the Dp-brane with the gauge field excited from the p'-p' open string. The vertex operator of the gauge field is

$$\mathcal{V}_A(\xi; k_M; \zeta_M) = i : \zeta_M(k)\dot{X}^M(\xi)\,\exp(ik_NX^N)(\xi) : \ , \tag{3.15}$$

where $\dot{X}^M(\xi) \equiv (\partial+\overline{\partial})\,X^M(z,\overline{z})\big|_{z=\overline{z}=\xi}$. The physical state conditions for this vertex operator require the on-shell condition $\alpha'm^2 \equiv -\alpha'G^{MN}k_Mk_N = 0$ and the polarization condition $G^{MN}k_M\zeta_N = 0$.

Let us compute the three point disc amplitude

$$\mathcal{A}_3^{\mathrm{gauge}} = \tilde{c}\int\frac{d\xi^{(1)}d\xi^{(2)}d\xi^{(3)}}{d^3F(\xi^{(1)},\xi^{(2)},\xi^{(3)})}\langle 0|\mathcal{V}_{\mathrm{m}}^{(-)}(\xi^{(1)}; k_\mu^{(1)})\,\mathcal{V}_A(\xi^{(2)}; k_M^{(2)}; \zeta_M)\,\mathcal{V}_{\mathrm{n}}^{(+)}(\xi^{(3)}; k_\mu^{(3)})|0\rangle\ . \tag{3.16}$$

Following the calculation given in appendix C, we find that

$$\mathcal{A}_3^{\text{gauge}} = \tilde{c} \prod_{\mu=0}^{p} \delta(k_\mu^{(1)} + k_\mu^{(2)} + k_\mu^{(3)}) \prod_{1\le c<d\le 3} \exp\left[\frac{i}{2}\theta^{\mu\lambda} k_\mu^{(c)} k_\lambda^{(d)} \epsilon(\tau^{(c)} - \tau^{(d)})\right] \times$$
$$\times \left(\frac{2(1-\nu)}{\varepsilon}\right)^{\frac{\mathrm{m}+\mathrm{n}}{2}} \sqrt{\mathrm{m}!\,\mathrm{n}!}\;\alpha'\;\mathcal{K}_{\mathrm{nm}}\,, \tag{3.17}$$

where $\mathcal{K}_{\mathrm{nm}}$ is defined for the $\mathrm{m} \ge \mathrm{n}$ case as

$$\mathcal{K}_{\mathrm{n}\le\mathrm{m}} = \sqrt{\frac{\mathrm{n}!}{\mathrm{m}!}} \exp \mathcal{C}(\nu; k_m^{(2)}) \Bigg[\left\{ -(k^{(3)} - k^{(1)}) \underset{(p)}{\cdot} \zeta + (2\nu - 1) G^{Z\overline{Z}} (\kappa^{(2)}\overline{e} - \overline{\kappa}^{(2)} e) \right\} \times$$
$$\times \left(i\sqrt{2\Xi}\,\overline{\kappa}^{(2)}\right)^{\mathrm{m}-\mathrm{n}} L_{\mathrm{n}}^{(\mathrm{m}-\mathrm{n})}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) + 2G^{Z\overline{Z}} \times$$
$$\times (\kappa^{(2)}\overline{e} - \overline{\kappa}^{(2)} e) \left(i\sqrt{2\Xi}\,\overline{\kappa}^{(2)}\right)^{\mathrm{m}-\mathrm{n}} L_{\mathrm{n}-1}^{(\mathrm{m}-\mathrm{n}+1)} \times$$
$$\times \left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) - i(\mathrm{m}-\mathrm{n}) \frac{1-\nu}{\alpha'} \overline{e} \times$$
$$\times \sqrt{2\Xi} \left(i\sqrt{2\Xi}\,\overline{\kappa}^{(2)}\right)^{\mathrm{m}-\mathrm{n}-1} L_{\mathrm{n}}^{(\mathrm{m}-\mathrm{n})}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) \Bigg]\,, \tag{3.18}$$

and for the $\mathrm{m} < \mathrm{n}$ case as

$$\mathcal{K}_{\mathrm{n}>\mathrm{m}} = \sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}} \exp \mathcal{C}(\nu; k_m^{(2)}) \Bigg[\left\{ -(k^{(3)} - k^{(1)}) \underset{(p)}{\cdot} \zeta + (2\nu - 1) G^{Z\overline{Z}} (\kappa^{(2)}\overline{e} - \overline{\kappa}^{(2)} e) \right\} \times$$
$$\times \left(i\sqrt{2\Xi}\,\kappa^{(2)}\right)^{\mathrm{n}-\mathrm{m}} L_{\mathrm{m}}^{(\mathrm{n}-\mathrm{m})}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) + 2G^{Z\overline{Z}} \times$$
$$\times (\kappa^{(2)}\overline{e} - \overline{\kappa}^{(2)} e) \left(i\sqrt{2\Xi}\,\kappa^{(2)}\right)^{\mathrm{n}-\mathrm{m}} L_{\mathrm{m}-1}^{(\mathrm{n}-\mathrm{m}+1)} \times$$
$$\times \left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) + i(\mathrm{n}-\mathrm{m}) \frac{(1-\nu)}{\alpha'} e\sqrt{2\Xi} \times$$
$$\times \left(i\sqrt{2\Xi}\,\kappa^{(2)}\right)^{\mathrm{n}-\mathrm{m}-1} L_{\mathrm{m}}^{(\mathrm{n}-\mathrm{m})}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) \Bigg]\,. \tag{3.19}$$

Here the symbol "$\underset{(p)}{\cdot}$" denotes the inner product of the $p+1$ dimensional vectors along the Dp-brane worldvolume with respect to the open string metric: $A \underset{(p)}{\cdot} B = G^{\mu\lambda} A_\mu B_\lambda$, and e and $\overline{e}$ are the polarization tensors $\zeta_{p+1}(k_M)$ and $\zeta_{p+2}(k_M)$ in the complex notation defined as

$$e(k_M) \equiv \frac{1}{2}\left(\zeta_{p+1}(k_M) - i\zeta_{p+2}(k_M)\right)\,, \quad \overline{e}(k_M) \equiv \frac{1}{2}\left(\zeta_{p+1}(k_M) + i\zeta_{p+2}(k_M)\right)\,. \tag{3.20}$$

By using the formula for the Laguerre polynomial

$$L_{\mathrm{n}}^{(\beta)}(x) = \sum_{\mathrm{k}=0}^{\mathrm{n}} \frac{\Gamma(\mathrm{n}-\mathrm{k}+\beta-\alpha)}{\Gamma(\mathrm{n}-\mathrm{k}+1)\Gamma(\beta-\alpha)} L_{\mathrm{k}}^{(\alpha)}(x)\,, \tag{3.21}$$

we can recast the factor $\mathcal{K}_{\mathrm{nm}}$ as

$$\mathcal{K}_{\mathrm{n\leq m}} \equiv \left\{-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta+(2\nu-1)G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\right\}\mathcal{I}_{\mathrm{nm}}+ \\ +2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\sqrt{\frac{\mathrm{n}!}{\mathrm{m}!}}\sum_{\mathrm{k=0}}^{\mathrm{n-1}}\sqrt{\frac{(\mathrm{m-n+k})!}{\mathrm{k}!}}\mathcal{I}_{\mathrm{k\ m-n+k}}- \\ -i(\mathrm{m-n})\frac{1-\nu}{\alpha'}\overline{e}\sqrt{2\Xi}\sqrt{\frac{\mathrm{n}!}{\mathrm{m}!}}\sum_{\mathrm{k=0}}^{\mathrm{n}}\sqrt{\frac{(\mathrm{m-n-1+k})!}{\mathrm{k}!}}\mathcal{I}_{\mathrm{k\ m-n-1+k}}\,, \quad (3.22)$$

and

$$\mathcal{K}_{\mathrm{n>m}} \equiv \left\{-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta+(2\nu-1)G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\right\}\mathcal{I}_{\mathrm{nm}}+ \\ +2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}}\sum_{\mathrm{k=0}}^{\mathrm{m-1}}\sqrt{\frac{(\mathrm{n-m+k})!}{\mathrm{k}!}}\mathcal{I}_{\mathrm{n-m+k\ k}}- \\ -i(\mathrm{n-m})\frac{1-\nu}{\alpha'}e\sqrt{2\Xi}\sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}}\sum_{\mathrm{k=0}}^{\mathrm{m}}\sqrt{\frac{(\mathrm{n-m-1+k})!}{\mathrm{k}!}}\mathcal{I}_{\mathrm{n-m-1+k\ k}}\,, \quad (3.23)$$

where $\mathcal{I}_{\mathrm{nm}}$ is the momentum dependent factor defined in eq.(3.9) and $\sum_{\mathrm{k=0}}^{\mathrm{n-1}}$ means zero if $\mathrm{n}=0$.

The first line in each of eqs. (3.22) and (3.23) has a similar feature to the scattering amplitude $\mathcal{A}_3^{\mathrm{tachyon}}$ in the sense that it can be obtained from the amplitude of the $\mathrm{m}=\mathrm{n}=0$ case by replacing the gaussian damping factor $\exp\mathcal{C}(\nu;k_m^{(2)})$ with $\mathcal{I}_{\mathrm{nm}}(b_{(p+2)/2},k_m^{(2)})$. The second and the third lines in each of eqs. (3.22) and (3.23) originate in the interactions between the almost-zero-modes and the gauge field on D$(p+2)$-brane worldvolume.

In the case of $\mathrm{m}=\mathrm{n}$, $\mathcal{K}_{\mathrm{mm}}$ has a simple form:

$$\mathcal{K}_{\mathrm{mm}} = \left\{-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta+(2\nu-1)G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\right\}\mathcal{I}_{\mathrm{mm}}+ \\ +2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}-\overline{\kappa}^{(2)}e)\sum_{\mathrm{k=0}}^{\mathrm{m-1}}\mathcal{I}_{\mathrm{kk}}\,. \quad (3.24)$$

In the zero slope limit, the multiplicative factors $\mathcal{I}_{\mathrm{mm}}$ and $\sum_{\mathrm{k=0}}^{\mathrm{m-1}}\mathcal{I}_{\mathrm{kk}}$ in this equation become projection operators $|\mathrm{m}\rangle\langle\mathrm{m}|$ and $\sum_{\mathrm{k=0}}^{\mathrm{m-1}}|\mathrm{k}\rangle\langle\mathrm{k}|$ respectively. These are what we desired to obtain.

4. Three point amplitudes and multiplicative factors in superstring theory

In this section we extend the analyses in the last section into the superstring case. We study the process in which the Dp-brane is probed with the noncommutative gauge field on the D$(p+2)$-brane worldvolume.

4.1 Vertex operators of light states of p-$(p+2)$ open string

We would like to obtain the physical state of the p-$(p+2)$ open string which is excited by almost-zero-modes $\overline{\alpha}_{-1+\nu}$ and survives the zero-slope limit (2.13). Let us consider the state

$$\zeta_\mu(k_\lambda) b^\mu_{-\frac{1}{2}} |0; k_\lambda\rangle \otimes (\overline{\alpha}_{-1+\nu})^{\mathrm{n}}\, \overline{b}_{-\frac{1}{2}+\nu} |\sigma, s\rangle\,, \tag{4.1}$$

where ζ_μ is the polarization tensor of this state and $|0; k_\lambda\rangle$ is the momentum eigenstate in the $x^0, \ldots, x^p$ directions. In the above we have divided the state into two pieces; the former comes from the string coordinates $\mathbb{X}^\mu$ and the latter from $\mathbb{Z}$ and $\overline{\mathbb{Z}}$. We impose on this state the conditions

$$\alpha' m^2 \equiv -\alpha' G^{\mu\lambda} k_\mu k_\lambda = \left(\mathrm{n} + \frac{1}{2}\right)(1-\nu)\,, \quad G^{\mu\lambda} k_\mu \zeta_\lambda = 0\,. \tag{4.2}$$

We find that, owing to these conditions, the state (4.1) satisfies the physical state conditions in the (-1)-picture. The on-shell condition in eq. (4.2) implies that this state remains light in the zero-slope limit (2.13). The state (4.1) is therefore what we wanted.

We denote the vertex operators whose lower component fields correspond to the state (4.1) and its conjugate state by

$$\begin{aligned} \mathbb{U}^{(+)}_{\mathrm{n}}(\xi, \theta; k_\mu; \zeta_\mu) &= \mathcal{U}^{(+,-1)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu) + \theta\, \mathcal{U}^{(+,0)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu)\,, \\ \mathbb{U}^{(-)}_{\mathrm{n}}(\xi, \theta; k_\mu; \zeta_\mu) &= \mathcal{U}^{(-,-1)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu) + \theta\, \mathcal{U}^{(-,0)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu)\,, \end{aligned} \tag{4.3}$$

respectively. The lower component fields $\mathcal{U}^{(\pm,-1)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu)$ take the forms of

$$\mathcal{U}^{(\pm,-1)}_{\mathrm{n}}(\xi; k_\mu; \zeta_\mu) = :\frac{1}{2}\zeta_\mu(k_\lambda)\left(\psi^\mu + \widetilde{\psi}^\mu\right) \exp\left(i k_\rho X^\rho\right)(\xi) : \, U^{(\pm,-1)}_{\mathrm{n}}(\xi)\,, \tag{4.4}$$

where $U^{(\pm,-1)}_{\mathrm{n}}(\xi)$ are the contributions from the fields $\mathbb{Z}$ and $\overline{\mathbb{Z}}$ defined by

$$\begin{aligned} \lim_{\xi\to 0} U^{(+,-1)}_{\mathrm{n}}(\xi)\,|0\rangle &= (\overline{\alpha}_{-1+\nu})^{\mathrm{n}}\, \overline{b}_{-\frac{1}{2}+\nu}\, |\sigma, s\rangle\,, \\ \lim_{\xi\to\infty} \xi^{2h} \langle 0|\, U^{(-,-1)}_{\mathrm{n}}(\xi) &= \langle \sigma, s|\, (\alpha_{1-\nu})^{\mathrm{n}}\, b_{\frac{1}{2}-\nu}\,, \end{aligned} \tag{4.5}$$

and h is the conformal weight of $U^{(\pm,-1)}_{\mathrm{n}}(\xi)$: $h = \left(\mathrm{n} + \frac{1}{2}\right)(1-\nu)$.

4.2 Three point amplitude

In superstring theory the gauge field emission vertex operator on the D$(p+2)$-brane worldvolume is

$$\mathbb{V}_A(\xi, \theta; k_M; \zeta_M) = :\frac{i}{2}\zeta_M(k)\dot{\mathbb{X}}^M \exp\left[i\sqrt{\frac{\alpha'}{2}} k_N \mathbb{X}^N\right](\xi, \theta) :\,, \tag{4.6}$$

where $\dot{\mathbb{X}}^M(\xi,\theta) \equiv (D+\overline{D})\mathbb{X}^M(\mathbf{z},\overline{\mathbf{z}})\big|_{z=\overline{z}=\xi,\theta=\overline{\theta}}$ and $D=\frac{\partial}{\partial\theta}+\theta\frac{\partial}{\partial z}$ and $\overline{D}=\frac{\partial}{\partial\overline{\theta}}+\overline{\theta}\frac{\partial}{\partial\overline{z}}$ are the superspace covariant derivatives. The physical state conditions for this vertex operator is the same as those in the bosonic string case.

We evaluate the three point disc amplitude

$$\mathcal{A}_3 = c' \int \frac{\prod_{a=1}^3 d\xi^{(a)} d\theta^{(a)}}{V_{\rm SCKV}} \langle 0|\, \mathbb{U}_{\rm m}^{(-)}(\xi^{(1)},\theta^{(1)};k_\mu^{(1)};\zeta_\mu^{(1)})\, \mathbb{V}_A(\xi^{(2)},\theta^{(2)};k_M^{(2)};\zeta_M^{(2)}) \times \\ \times\ \mathbb{U}_{\rm n}^{(+)}(\xi^{(3)},\theta^{(3)};k_\mu^{(3)};\zeta_\mu^{(3)})\,|0\rangle\,, \tag{4.7}$$

where $V_{\rm SCKV}$ stands for the gauge volume of the graded $\mathrm{SL}(2,\mathbb{R})$ group generated by the superconformal Killing vectors on the super worldsheet. To fix the odd elements of the gauge degrees of freedom, we set $\theta^{(1)}=\theta^{(3)}=0$. This gauge choice amounts to factoring out the following volume element from the integration in eq.(4.7):

$$d^3F(\xi^{(1)},\xi^{(2)},\xi^{(3)})\, d\theta^{(1)}d\theta^{(3)}\,(\xi^{(1)}-\xi^{(3)})\,. \tag{4.8}$$

Thus the three point amplitude (4.7) turns out to be

$$\mathcal{A}_3 = c'(\xi^{(2)}-\xi^{(3)})(\xi^{(3)}-\xi^{(1)}) \int d\theta^{(2)} \langle 0|\mathcal{U}_{\rm m}^{(-,-1)}(\xi^{(1)};k_\mu^{(1)};\zeta_\mu^{(1)})\mathbb{V}_A(\xi^{(2)},\theta^{(2)};k_M^{(2)};\zeta_M^{(2)}) \times \\ \times\ \mathcal{U}_{\rm n}^{(+,-1)}(\xi^{(3)};k_\mu^{(3)};\zeta_\mu^{(3)})\,|0\rangle\,. \tag{4.9}$$

As a result of the calculation presented in appendix D, we find that the amplitude $\mathcal{A}_3$ becomes

$$\mathcal{A}_3 = c' \prod_{\mu=0}^{p} \delta(k_\mu^{(1)}+k_\mu^{(2)}+k_\mu^{(3)}) \prod_{1\le c<d\le 3} \exp\left[\frac{i}{2}\theta^{\mu\nu}k_\mu^{(c)}k_\lambda^{(d)}\epsilon(\tau^{(c)}-\tau^{(d)})\right] \times \\ \times \sqrt{\rm m!n!}\left(\frac{2(1-\nu)}{\varepsilon}\right)^{\frac{\rm m+n}{2}} \frac{2}{\varepsilon}\sqrt{\frac{\alpha'}{2}}\,\mathcal{M}_{\rm nm}\,, \tag{4.10}$$

where the factor $\mathcal{M}_{\rm nm}$ is defined for the $\rm m \ge n$ case as

$$\begin{aligned}
\mathcal{M}_{\rm n\le m} = &\sqrt{\frac{\rm n!}{\rm m!}}\exp\mathcal{C}(\nu;k_m^{(2)}) \times \\
&\times\Bigg[-2\left\{(k^{(2)}\underset{(p)}{\cdot}\zeta^{(1)})(\zeta^{(2)}\underset{(p)}{\cdot}\zeta^{(3)}) + (k^{(1)}\underset{(p)}{\cdot}\zeta^{(3)})(\zeta^{(1)}\underset{(p)}{\cdot}\zeta^{(2)})\right\} \times \\
&\times \left(i\sqrt{2\Xi}\overline{\kappa}^{(2)}\right)^{\rm m-n} L_{\rm n}^{(\rm m-n)}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) + (\zeta^{(3)}\underset{(p)}{\cdot}\zeta^{(1)}) \times \\
&\times\Bigg\{\left(-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta^{(2)} + G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)})\right) \times \\
&\times \left(i\sqrt{2\Xi}\overline{\kappa}^{(2)}\right)^{\rm m-n} L_{\rm n}^{(\rm m-n)}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) + 2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)}) \times \\
&\times \left(i\sqrt{2\Xi}\overline{\kappa}^{(2)}\right)^{\rm m-n} L_{\rm n-1}^{(\rm m-n+1)}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right) - i(\rm m-n) \times \\
&\times \sqrt{2\Xi}\frac{1-\nu}{\alpha'}\overline{e}^{(2)}\left(i\sqrt{2\Xi}\overline{\kappa}^{(2)}\right)^{\rm m-n-1} L_{\rm n}^{(\rm m-n)}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right)\Bigg\}\Bigg]\,,
\end{aligned} \tag{4.11}$$

and for the m < n case as

$$
\begin{aligned}
\mathcal{M}_{\mathrm{n}\geq\mathrm{m}} = & \sqrt{\frac{\mathrm{m}!}{\mathrm{n}!}}\exp\mathcal{C}(\nu;k^{(2)}_m)\times \\
& \times\Big[-2\Big\{(k^{(2)}\underset{(p)}{\cdot}\zeta^{(1)})(\zeta^{(2)}\underset{(p)}{\cdot}\zeta^{(3)})+(k^{(1)}\underset{(p)}{\cdot}\zeta^{(3)})(\zeta^{(1)}\underset{(p)}{\cdot}\zeta^{(2)})\Big\}\times \\
& \quad\times\left(i\sqrt{2\Xi}\kappa^{(2)}\right)^{\mathrm{n-m}}L^{(\mathrm{n-m})}_{\mathrm{m}}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right)+(\zeta^{(3)}\underset{(p)}{\cdot}\zeta^{(1)})\times \\
& \quad\times\Big\{\left(-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta^{(2)}+G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)})\right)\times \\
& \qquad\times\left(i\sqrt{2\Xi}\kappa^{(2)}\right)^{\mathrm{n-m}}L^{(\mathrm{n-m})}_{\mathrm{n}}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right)+2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)})\times \\
& \qquad\times\left(i\sqrt{2\Xi}\kappa^{(2)}\right)^{\mathrm{n-m}}L^{(\mathrm{n-m+1})}_{\mathrm{m-1}}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right)+i(\mathrm{n-m})\sqrt{2\Xi}\frac{1-\nu}{\alpha'}\times \\
& \qquad\times e^{(2)}\left(i\sqrt{2\Xi}\kappa^{(2)}\right)^{\mathrm{n-m-1}}L^{(\mathrm{n-m})}_{\mathrm{m}}\left(2\Xi\kappa^{(2)}\overline{\kappa}^{(2)}\right)\Big\}\Big].
\end{aligned}
\tag{4.12}
$$

Comparing the above equations with eqs. (3.18) and (3.19), we find that the factor $\mathcal{M}_{\mathrm{nm}}$ consists of terms which quite resemble $\mathcal{K}_{\mathrm{nm}}$. We can therefore rewrite the above equations into the form involving the factors $\mathcal{I}_{\mathrm{nm}}$ as we carried out in eqs. (3.22) and (3.23). Here we focus on $\mathcal{M}_{\mathrm{mm}}$ in particular:

$$
\begin{aligned}
\mathcal{M}_{\mathrm{mm}} = & -2\Big\{(k^{(2)}\underset{(p)}{\cdot}\zeta^{(1)})(\zeta^{(2)}\underset{(p)}{\cdot}\zeta^{(3)})+(k^{(1)}\underset{(p)}{\cdot}\zeta^{(3)})(\zeta^{(1)}\underset{(p)}{\cdot}\zeta^{(2)})\Big\}\mathcal{I}_{\mathrm{mm}}+ \\
& +(\zeta^{(3)}\underset{(p)}{\cdot}\zeta^{(1)})\Big\{\left(-(k^{(3)}-k^{(1)})\underset{(p)}{\cdot}\zeta^{(2)}+G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)})\right)\mathcal{I}_{\mathrm{mm}}+ \\
& \qquad +2G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}-\overline{\kappa}^{(2)}e^{(2)})\sum_{\mathrm{k}=0}^{\mathrm{m}-1}\mathcal{I}_{\mathrm{kk}}\Big\}.
\end{aligned}
\tag{4.13}
$$

From this equation, we find that in the case of m = n the amplitude involves the multiplicative factors $\mathcal{I}_{\mathrm{mm}}$ and $\sum_{\mathrm{k}=0}^{\mathrm{m}-1}\mathcal{I}_{\mathrm{kk}}$ which become projection operators $|\mathrm{m}\rangle\langle\mathrm{m}|$ and $\sum_{\mathrm{k}=0}^{\mathrm{m}-1}|\mathrm{k}\rangle\langle\mathrm{k}|$ respectively in the zero-slope limit. These are what we desired.

5. Conclusion and discussions

In this paper we have calculated three point scattering amplitudes for processes probing the Dp-brane with the p'-p' open string in the p-p' system with B field. We carried out the calculation in bosonic string theory and in superstring theory. We have focused on processes in which the almost-zero-modes of the p-p' open string are involved. Following ref. [13], we have read off momentum dependent multiplicative factors from three point disc amplitudes for such processes. We have showed that in the case of m = n these multiplicative factors take such forms that become Weyl transforms of projection operators in the zero-slope limit.

We think that this result provides further evidence for the observation that the tower of the light states surviving the zero slope limit [8, 22] play a role to realize the D-branes as noncommutative solitons on the higher dimensional noncommutative D-brane worldvolume.

The fact that many light states survive the zero-slope limit is a particular result of a nonvanishing B field background. Thus it is expected that the tower of light states cause the behavior of D-branes within a noncommutative D-brane to look quite different from that within a commutative D-brane. The roles played by the tower of the light states in this system seem to need more investigation and clarification. As well as the perturbative string analysis, the field theoretical analysis is expected to be important and useful in some aspect of this problem.

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A. Weyl correspondence

There is a one-to-one correspondence, referred to as the Weyl correspondence, between a function on the commutative $\mathbb{R}^2$ space and an operator on noncommutative $\mathbb{R}^2$ space (see e.g. [15, 26]). In order to fix the notation, we would like to give some formulae of Weyl ordering prescription.

Let us consider the noncommutative $\mathbb{R}^2$ space characterized by[1]

$$[\hat{x}^1, \hat{x}^2] = -i\theta^{12} \ , \quad \theta^{12} \in \mathbb{R} \, . \tag{A.1}$$

This allows us to regard the noncommutative $\mathbb{R}^2$ space as the single particle Hilbert space $\mathcal{H}$ of quantum mechanics. Given a function $f(x^1, x^2)$ on the commutative $\mathbb{R}^2$ space, we uniquely determine a corresponding operator $\widehat{\mathcal{O}}_f(\hat{x}^1, \hat{x}^2)$ on the noncommutative $\mathbb{R}^2$ space by the Weyl ordering prescription:

$$\begin{aligned} \widehat{\mathcal{O}}_f(\hat{x}^1, \hat{x}^2) &\equiv \int \frac{dk_1 dk_2}{(2\pi)^2} \widehat{V}(k_1, k_2) \widetilde{f}(k_1, k_2) \, , \\ \widehat{V}(k_1, k_2) &\equiv \exp\left[-i\left(k_1 \hat{x}^1 + k_2 \hat{x}^2\right)\right] \, , \end{aligned} \tag{A.2}$$

[1] The coordinates x^1 and x^2 in this appendix correspond to x^{p+1} and x^{p+2} in the main part of this paper respectively. The minus sign on the r.h.s. in eq. (A.1) is necessary because we consider the $\sigma = \pi$ end of the string [6] (i.e. the negative real axis on the z-plane of the worldsheet). This is just a convention.

where $\widetilde{f}(k_1,k_2)$ is the Fourier transform of the function $f(x^1,x^2)$:

$$\widetilde{f}(k_1,k_2) \equiv \int dx^1 dx^2 e^{i\left(k_1x^1+k_2x^2\right)} f(x^1,x^2)\,. \tag{A.3}$$

Or equivalently,

$$\begin{aligned}\widehat{\mathcal{O}}_f(\hat{x}^1,\hat{x}^2) &= \int dx^1dx^2 \widehat{\triangle}(x^1,x^2) f(x^1,x^2)\,,\\ \widehat{\triangle}(x^1,x^2) &\equiv \int \frac{dk_1dk_2}{(2\pi)^2} \exp\left[-ik_1(\hat{x}^1-x^1)-ik_2(\hat{x}^2-x^2)\right]\;.\end{aligned} \tag{A.4}$$

The inverse transformation of eq.(A.4) is

$$\begin{aligned} f(x^1,x^2) &= |\theta^{12}| \int dk_2\, e^{-ik_2x^2} \left\langle x^1 - \frac{\theta^{12}}{2}k_2 \right| \widehat{\mathcal{O}}_f(\hat{x}^1,\hat{x}^2) \left| x^1 + \frac{\theta^{12}}{2}k_2 \right\rangle \\ &= |\theta^{12}| \int dk_1\, e^{-ik_1x^1} \left\langle x^2 + \frac{\theta^{12}}{2}k_1 \right| \widehat{\mathcal{O}}_f(\hat{x}^1,\hat{x}^2) \left| x^2 - \frac{\theta^{12}}{2}k_1 \right\rangle\;, \end{aligned} \tag{A.5}$$

where $|x^1\rangle$ and $|x^2\rangle$ are the eigenstates of the operators $\hat{x}^1$ and $\hat{x}^2$ respectively: $\hat{x}^1|x^1\rangle = x^1|x^1\rangle$, $\hat{x}^2|x^2\rangle = x^2|x^2\rangle$, normalized as $\langle x'^1|x^1\rangle = \delta(x'^1-x^1)$ and $\langle x'^2|x^2\rangle = \delta(x'^2-x^2)$. This can be proved by using the relations

$$\begin{aligned}\widehat{\triangle}(\hat{x}^1,\hat{x}^2) &= \int \frac{dk_2}{2\pi}\, e^{ik_2x^2} \left| x^1 - \frac{\theta^{12}}{2}k_2 \right\rangle \left\langle x^1 + \frac{\theta^{12}}{2}k_2 \right| \\ &= \int \frac{dk_1}{2\pi}\, e^{ik_1x^1} \left| x^2 + \frac{\theta^{12}}{2}k_1 \right\rangle \left\langle x^2 - \frac{\theta^{12}}{2}k_1 \right|\,. \end{aligned} \tag{A.6}$$

It is well-known that this correspondence relation maps the product of the operators into the $*$-product of the functions: $\widehat{\mathcal{O}}_f\widehat{\mathcal{O}}_g = \widehat{\mathcal{O}}_{f*g}$, where

$$f*g(x^1,x^2) \equiv f(x^1,x^2) e^{-i\theta^{mn}\overleftarrow{\partial}_m\overrightarrow{\partial}_n} g(x^1,x^2)\,. \tag{A.7}$$

The formula

$$\mathrm{Tr}_{\mathcal{H}}\left(\widehat{V}^\dagger(u_1,u_2)\widehat{V}(k_1,k_2)\right) = \frac{2\pi}{|\theta^{12}|}\delta(u_1-k_1)\delta(u_2-k_2) \tag{A.8}$$

and eq. (A.2) yield

$$\widetilde{f}(k_1,k_2) = 2\pi|\theta^{12}|\; \mathrm{Tr}_{\mathcal{H}}\left(\widehat{V}^\dagger(k_1,k_2)\widehat{\mathcal{O}}_f(\hat{x}^1,\hat{x}^2)\right)\;. \tag{A.9}$$

Let us define the creation operator $\hat{a}^\dagger$ and the annihilation operator $\hat{a}$. We consider the case[2] in which $\theta^{12} < 0$ and $[\hat{x}^1,\hat{x}^2] = i|\theta^{12}|$. This is the situation

[2]For the case where $\theta^{12} > 0$, we should exchange the space indices 1 and 2 in the subsequent equations. For example, $\hat{a}$ and $\hat{a}^\dagger$ would be defined as $\hat{a} \equiv \frac{\hat{x}^2+i\hat{x}^1}{\sqrt{2|\theta^{12}|}}$, $\hat{a}^\dagger \equiv \frac{\hat{x}^2-i\hat{x}^1}{\sqrt{2|\theta^{12}|}}$.

analyzed in the main part of this paper (see eq. (2.14)). In this case we define $\hat{a}$ and $\hat{a}^\dagger$ as

$$\hat{a} \equiv \frac{\hat{x}^1 + i\hat{x}^2}{\sqrt{2|\theta^{12}|}}\,, \qquad \hat{a}^\dagger \equiv \frac{\hat{x}^1 - i\hat{x}^2}{\sqrt{2|\theta^{12}|}}\,. \tag{A.10}$$

Let $f_{\rm nm}(x^1, x^2)$ be the Weyl transform of the operator $|{\rm n}\rangle\langle{\rm m}|$ $({\rm m}, {\rm n} = 0, 1, 2, \ldots)$, where $|{\rm m}\rangle$ are the harmonic oscillator eigenstates: $|{\rm m}\rangle \equiv \frac{(\hat{a}^\dagger)^{\rm m}}{\sqrt{{\rm m}!}}|0\rangle$. From eq. (A.9) one can find that the Fourier transform $\widetilde{f}_{\rm nm}(k_1, k_2)$ of the function $f_{\rm nm}(x^1, x^2)$ takes the form of

$$\widetilde{f}_{\rm nm}(k_1, k_2) = 2\pi|\theta^{12}|\ {\rm Tr}_{\mathcal{H}}\left(\widehat{V}^\dagger(k_1, k_2)|{\rm n}\rangle\langle{\rm m}|\right) = 2\pi|\theta^{12}|\,\langle{\rm m}|\widehat{V}^\dagger(k_1, k_2)|{\rm n}\rangle\,. \tag{A.11}$$

We can rewrite the operator $\widehat{V}^\dagger(k_1, k_2)$ as

$$\widehat{V}^\dagger(k_1, k_2) = e^{i\sqrt{2|\theta^{12}|}\,\left(\kappa\hat{a}+\overline{\kappa}\hat{a}^\dagger\right)} = e^{-|\theta^{12}|\kappa\overline{\kappa}}\, e^{i\sqrt{2|\theta^{12}|}\,\overline{\kappa}\hat{a}^\dagger}\, e^{i\sqrt{2|\theta^{12}|}\,\kappa\hat{a}}\,, \tag{A.12}$$

where $\kappa = \frac{1}{2}(k_1 - ik_2)$ and $\overline{\kappa} = \frac{1}{2}(k_1 + ik_2)$. This yields

$$\widetilde{f}_{\rm nm}(k_1, k_2) = \tag{A.13}$$
$$= \begin{cases} 2\pi|\theta^{12}|\sqrt{\dfrac{{\rm n}!}{{\rm m}!}}\left(i\sqrt{2|\theta^{12}|}\,\overline{\kappa}\right)^{\rm m-n} e^{-|\theta^{12}|\kappa\overline{\kappa}} L_{\rm n}^{(\rm m-n)}\left(2|\theta^{12}|\kappa\overline{\kappa}\right) & ({\rm m} \geq {\rm n}) \\ 2\pi|\theta^{12}|\sqrt{\dfrac{{\rm m}!}{{\rm n}!}}\left(i\sqrt{2|\theta^{12}|}\,\kappa\right)^{\rm n-m} e^{-|\theta^{12}|\kappa\overline{\kappa}} L_{\rm m}^{(\rm n-m)}\left(2|\theta^{12}|\kappa\overline{\kappa}\right) & ({\rm m} < {\rm n}) \end{cases},$$

where $L_{\rm n}^{(\alpha)}(x)$ is the Laguerre polynomial (3.11).

B. Calculation of $\mathcal{A}_3^{\rm tachyon}$

In this appendix we give some details in the calculation of eq. (3.8).

The location $\xi^{(2)}$ of the tachyon vertex operator $\mathcal{V}_\phi$ is restricted to the interval $\xi^{(2)} \in [\xi^{(1)}, \xi^{(3)}]$ as explained. Using the $SL(2,\mathbb{R})$ gauge degrees of freedom on the worldsheet, we choose $\xi^{(1)} = -\infty$ and $\xi^{(3)} = 0$, so that the negative real axis becomes the worldsheet boundary attaching on the D$(p+2)$-brane. The correlation function in the integrand on the r.h.s. in eq. (3.6) is factorized into two pieces; the one is the contribution from $X^0, \ldots, X^p$ and the other is form Z and $\overline{Z}$:

$$\begin{aligned} &\langle 0|\mathcal{V}_{\rm m}^{(-)}(\xi^{(1)}; k_\mu^{(1)})\,\mathcal{V}_\phi(\xi^{(2)}; k_M^{(2)})\,\mathcal{V}_{\rm n}^{(+)}(\xi^{(3)}; k_\mu^{(3)})|0\rangle = \\ &\quad = \langle 0|:\exp\left[ik_\mu^{(1)}X^\mu\right](\xi^{(1)})::\exp\left[ik_\mu^{(2)}X^\mu\right](\xi^{(2)})::\exp\left[ik_\mu^{(3)}X^\mu\right](\xi^{(3)}):|0\rangle \times \\ &\qquad \times \langle 0|V_{\rm m}^{(-)}(\xi^{(1)})\,V_\phi(\xi^{(2)}; k_m^{(2)})\,V_{\rm n}^{(+)}(\xi^{(3)})|0\rangle\,. \end{aligned} \tag{B.1}$$

The propagator (2.4) leads us to find that the former piece becomes [23, 24, 8]

$$\begin{aligned}
&\langle 0| : \exp\left[ik_\mu^{(1)}X^\mu\right](\xi^{(1)}) :: \exp\left[ik_\mu^{(2)}X^\mu\right](\xi^{(2)}) :: \exp\left[ik_\mu^{(3)}X^\mu\right](\xi^{(3)}) : |0\rangle = \\
&= \prod_{\mu=0}^{p} \delta(k_\mu^{(1)} + k_\mu^{(2)} + k_\mu^{(3)}) \times \\
&\quad \times \prod_{1\le c<d\le 3} (x^{(c)} - x^{(d)})^{2\alpha' G^{\mu\nu} k_\mu^{(c)} k_\nu^{(d)}} \exp\left[\frac{i}{2}\theta^{\mu\nu} k_\mu^{(c)} k_\nu^{(d)} \epsilon(x^{(c)} - x^{(d)})\right] . \qquad \text{(B.2)}
\end{aligned}$$

Here we have introduced positive real variables $x^{(c)} \equiv -\xi^{(c)} = e^{\tau^{(c)}}$ $(c = 1, 2, 3)$. Since we have chosen $x^{(1)} = \infty$ and $x^{(3)} = 0$, the latter piece on the r.h.s. of eq.(B.1) becomes

$$\begin{aligned}
&\langle 0| V_{\mathrm{m}}^{(-)}(-x^{(1)})\, V_\phi(-x^{(2)}; k_m^{(2)})\, V_{\mathrm{n}}^{(+)}(-x^{(3)}) |0\rangle = \\
&= \left(x^{(1)}\right)^{-(2\mathrm{m}+\nu)(1-\nu)} \left(x^{(2)}\right)^{-2\alpha' G^{Z\overline{Z}} \kappa^{(2)} \overline{\kappa}^{(2)}} \exp \mathcal{C}\left(\nu; k_m^{(2)}\right) \times \\
&\quad \times \langle \sigma | \left(\alpha_{1-\nu}\right)^{\mathrm{m}} \circ\!\!\circ \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)\right](-x^{(2)}) \circ\!\!\circ \left(\overline{\alpha}_{-1+\nu}\right)^{\mathrm{n}} |\sigma\rangle \ , \qquad \text{(B.3)}
\end{aligned}$$

with $x^{(1)} = \infty$ and $x^{(3)} = 0$, where $\mathcal{C}(\nu; k_m)$ is defined in eq. (3.12). Here we have used the defining relations (3.2) and applied the renormal ordering formula (2.11) to the operator $V_\phi(-x^{(2)}; k_m^{(2)})$:

$$V_\phi(-x^{(2)}; k_m^{(2)}) = \left(x^{(2)}\right)^{-2\alpha' G^{Z\overline{Z}} \kappa^{(2)} \overline{\kappa}^{(2)}} \exp \mathcal{C}(\nu; k_m^{(2)}) \circ\!\!\circ \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)\right](-x^{(2)}) \circ\!\!\circ . \qquad \text{(B.4)}$$

Now that the operator $\circ\!\!\circ \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)\right](-x^{(2)}) \circ\!\!\circ$ is evaluated on the oscillator vacuum $|\sigma\rangle$, the fields Z and $\overline{Z}$ should be expanded as is described in eq.(2.6) and the normal ordering defined on $|\sigma\rangle$ in [13] be taken. Thus, by using the commutation relation (2.8), we obtain

$$\begin{aligned}
\left[\overline{\alpha}_{-1+\nu} \ , \ \circ\!\!\circ\, e^{i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)}(-x^{(2)}) \circ\!\!\circ\right] &= f\left(\nu; x^{(2)}; \kappa^{(2)}\right) \circ\!\!\circ\, e^{i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)}(-x^{(2)}) \circ\!\!\circ , \\
\left[\alpha_{1-\nu} \ , \ \circ\!\!\circ\, e^{i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)}(-x^{(2)}) \circ\!\!\circ\right] &= g\left(\nu; x^{(2)}; \overline{\kappa}^{(2)}\right) \circ\!\!\circ\, e^{i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)}(-x^{(2)}) \circ\!\!\circ , \qquad \text{(B.5)}
\end{aligned}$$

where

$$\begin{aligned}
f\left(\nu; x^{(2)}; \kappa^{(2)}\right) &\equiv -\left(x^{(2)}\right)^{-1+\nu} i\kappa^{(2)} \sqrt{\frac{\alpha'}{2}\frac{4}{\varepsilon}} \sin(\pi\nu) \, , \\
g\left(\nu; x^{(2)}; \overline{\kappa}^{(2)}\right) &\equiv \left(x^{(2)}\right)^{1-\nu} i\overline{\kappa}^{(2)} \sqrt{\frac{\alpha'}{2}\frac{4}{\varepsilon}} \sin(\pi\nu) \, . \qquad \text{(B.6)}
\end{aligned}$$

The commutation relations (B.5) and $[\alpha_{1-\nu}, \overline{\alpha}_{-1+\nu}] = \frac{2}{\varepsilon}(1-\nu)(\equiv q)$ yield

$$\begin{aligned}
&\left(\alpha_{1-\nu}\right)^{\mathrm{m}} \circ\!\!\circ \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)} \overline{Z}\right)\right](-x^{(2)}) \circ\!\!\circ \left(\overline{\alpha}_{-1+\nu}\right)^{\mathrm{n}} = \\
&= \begin{cases}
\displaystyle\sum_{l=0}^{\mathrm{n}} \frac{\mathrm{m}!\,\mathrm{n}!}{l!(\mathrm{n}-l)!\,(l+\mathrm{m}-\mathrm{n})!} q^{\mathrm{n}-l} \left(\overline{\alpha}_{-1+\nu} - f\right)^l \left(g + \alpha_{1-\nu}\right)^{\mathrm{m}-\mathrm{n}+l} & (\mathrm{m} \ge \mathrm{n}) \\
\displaystyle\sum_{l=0}^{\mathrm{m}} \frac{\mathrm{m}!\,\mathrm{n}!}{l!(\mathrm{m}-l)!\,(l+\mathrm{n}-\mathrm{m})!} q^{\mathrm{m}-l} \left(\overline{\alpha}_{-1+\nu} - f\right)^{l+(\mathrm{n}-\mathrm{m})} \left(g + \alpha_{1-\nu}\right)^l & (\mathrm{m} < \mathrm{n})
\end{cases} . \qquad \text{(B.7)}
\end{aligned}$$

From this we find that

$$
\begin{aligned}
&\langle\sigma|\left(\alpha_{1-\nu}\right)^{\mathrm{m}} \vdots \ \exp\left[i\left(\kappa^{(2)}Z+\overline{\kappa}^{(2)}\overline{Z}\right)\right]\left(-x^{(2)}\right) \vdots \left(\overline{\alpha}_{-1+\nu}\right)^{\mathrm{n}}|\sigma\rangle = \\
&\qquad = \begin{cases} \mathrm{n}!\, q^{\mathrm{n}} g^{\mathrm{m-n}}\, L_{\mathrm{n}}^{(\mathrm{m-n})}\left(\dfrac{fg}{q}\right) & (\mathrm{m}\geq \mathrm{n}) \\[2ex] \mathrm{m}!\, q^{\mathrm{m}}(-f)^{\mathrm{n-m}}\, L_{\mathrm{m}}^{(\mathrm{n-m})}\left(\dfrac{fg}{q}\right) & (\mathrm{m}<\mathrm{n}) \end{cases}\ ,
\end{aligned}
\tag{B.8}
$$

where $L_{\mathrm{n}}^{(\alpha)}(x)$ is the Laguerre polynomial (3.11). Substituting eqs. (B.2), (B.3) and (B.8) into eq. (B.1) and using the on-shell conditions, we obtain the three point amplitude (3.8).

C. Calculation of $\mathcal{A}_3^{\mathrm{gauge}}$

In this appendix we provide the steps to obtain eq. (3.17).

Introducing a parameter a, we rewrite the vertex operator into the following form [25]:

$$
\begin{aligned}
\mathcal{V}_A(\xi;k_M;\zeta_M) &= \frac{\partial}{\partial a} : \exp\left[i\left(k_M X^M + a\zeta_M \dot{X}^M\right)\right](\xi) : \Big|_{a=0} \\
&= \frac{\partial}{\partial a}\Big\{ : \exp\Big[i\left(k_\mu X^\mu + a\zeta_\mu \dot{X}^\mu\right)\Big] : \times \\
&\qquad\qquad \times : \exp\left[i\left(\kappa Z + \overline{\kappa}\overline{Z} + ae\dot{Z} + a\overline{e}\dot{\overline{Z}}\right)\right] : \Big\}\Big|_{a=0}\ ,
\end{aligned}
\tag{C.1}
$$

where e and $\overline{e}$ are defined in eq. (3.20). As was done in the appendix B, we divide the correlation function in the integrand on the r.h.s. in eq. (3.16) into two pieces; the contribution from X^μ ($\mu = 0,\ldots,p$) and that from Z and $\overline{Z}$:

$$
\begin{aligned}
&\langle 0|\mathcal{V}_{\mathrm{m}}^{(-)}(\xi^{(1)};k_\mu^{(1)})\,\mathcal{V}_A(\xi^{(2)};k_M^{(2)};\zeta_M)\,\mathcal{V}_{\mathrm{n}}^{(+)}(\xi^{(3)};k_\mu^{(3)})|0\rangle = \\
&= \frac{d}{da}\Big[\langle 0| : \exp\left(k_\mu^{(1)}X^\mu\right)(\xi^{(1)}) : : \exp\left[i\left(k_\mu^{(2)}X^\mu + a\zeta_\mu\dot{X}^\mu\right)\right](\xi^{(2)}) : \times \\
&\qquad \times : \exp\left(ik_\mu^{(3)}X^\mu\right)(\xi^{(3)}) : |0\rangle \times \\
&\quad \times\langle 0|V_{\mathrm{m}}^{(-)}(\xi^{(1)}) \ : \exp\Big[i\left(\kappa^{(2)}Z + \overline{\kappa}^{(2)}\overline{Z} + ae\dot{Z} + a\overline{e}\dot{\overline{Z}}\right)\Big](\xi^{(2)}) :\ V_{\mathrm{n}}^{(+)}(\xi^{(3)})|0\rangle\Big]_{a=0}.
\end{aligned}
\tag{C.2}
$$

The former piece in eq. (C.2) becomes [8, 23, 24, 25]

$$
\begin{aligned}
&\langle 0| : \exp\left(k^{(1)}_{\mu} X^{\mu}\right)(-x^{(1)}) : : \exp\left[i\left(k^{(2)}_{\mu} X^{\mu} + a\zeta_{\mu}\dot{X}^{\mu}\right)\right](-x^{(2)}) : \times \\
&\qquad\qquad\qquad\qquad \times\ : \exp\left(ik^{(3)}_{\mu} X^{\mu}\right)(-x^{(3)}) : |0\rangle = \\
&= \prod_{\mu=0}^{p} \delta(k^{(1)}_{\mu} + k^{(2)}_{\mu} + k^{(3)}_{\mu}) \exp\left[a\left(\frac{2\alpha' G^{\mu\nu} k^{(1)}_{\mu}\zeta_{\nu}}{x^{(1)} - x^{(2)}} - \frac{2\alpha' G^{\mu\nu} k^{(3)}_{\mu}\zeta_{\nu}}{x^{(2)} - x^{(3)}} \right)\right] \times \\
&\quad \times \prod_{1\le c<d\le 3} (x^{(c)} - x^{(d)})^{2\alpha' G^{\mu\nu} k^{(c)}_{\mu} k^{(d)}_{\nu}} \exp\left[\frac{i}{2}\theta^{\mu\nu} k^{(c)}_{\mu} k^{(d)}_{\nu} \epsilon(\tau^{(c)} - \tau^{(d)})\right] .
\end{aligned} \tag{C.3}
$$

Sending $x^{(1)} \to \infty$ and $x^{(3)} \to 0$, we find that the latter piece in eq.(C.2) takes the form of

$$
\begin{aligned}
&\langle 0|\, V^{(-)}_{\mathrm{m}}(-x^{(1)})\ : \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)}\overline{Z} + ae\dot{Z} + a\overline{e}\dot{\overline{Z}}\right)\right](-x^{(2)}) :\ V^{(+)}_{\mathrm{n}}(-x^{(3)})\,|0\rangle = \\
&= (x^{(1)})^{-(2\mathrm{m}+\nu)(1-\nu)} (x^{(2)})^{-2\alpha' G^{Z\overline{Z}}\kappa^{(2)}\overline{\kappa}^{(2)}} \times \\
&\times \exp\left[\mathcal{C}(\nu; k^{(2)}_{m}) + 2\alpha' G^{Z\overline{Z}} \left\{ a\left(\frac{\kappa^{(2)}\overline{e}\nu + \overline{\kappa}^{(2)} e(1-\nu)}{x^{(2)}} \right) + a^{2} e\overline{e}\frac{\nu(1-\nu)}{2(x^{(2)})^{2}} \right\}\right] \times \\
&\times \langle\sigma|(\alpha_{1-\nu})^{\mathrm{m}} \,\mathring{\circ}\, \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)}\overline{Z} + ae\dot{Z} + a\overline{e}\dot{\overline{Z}}\right)\right](-x^{(2)}) \,\mathring{\circ}\, (\overline{\alpha}_{-1+\nu})^{\mathrm{n}}|\sigma\rangle ,
\end{aligned} \tag{C.4}
$$

with $x^{(1)} = \infty$ and $x^{(3)} = 0$. Here we have applied the renormal ordering formula (2.11) to the gauge field vertex operator. In a similar way to the calculation of eq.(B.8), we obtain

$$
\begin{aligned}
&\langle\sigma|(\alpha_{1-\nu})^{\mathrm{m}} \,\mathring{\circ}\, \exp\left[i\left(\kappa^{(2)} Z + \overline{\kappa}^{(2)}\overline{Z} + a(e\dot{Z} + \overline{e}\dot{\overline{Z}})\right)\right](-x^{(2)}) \,\mathring{\circ}\, (\overline{\alpha}_{-1+\nu})^{\mathrm{n}}|\sigma\rangle = \\
&\qquad = \begin{cases} \mathrm{n}!\, q^{\mathrm{n}} \widetilde{g}^{\mathrm{m}-\mathrm{n}} L^{(\mathrm{m}-\mathrm{n})}_{\mathrm{n}}\left(\dfrac{\widetilde{f}\,\widetilde{g}}{q}\right) & (\mathrm{m} \ge \mathrm{n}) \\ \mathrm{m}!\, q^{\mathrm{m}} (-\widetilde{f})^{\mathrm{n}-\mathrm{m}} L^{(\mathrm{n}-\mathrm{m})}_{\mathrm{m}}\left(\dfrac{\widetilde{f}\,\widetilde{g}}{q}\right) & (\mathrm{m} < \mathrm{n}) \end{cases} ,
\end{aligned} \tag{C.5}
$$

where

$$
\begin{aligned}
\widetilde{f}(\nu; x^{(2)}; \kappa^{(2)}; e) &\equiv -(x^{(2)})^{-1+\nu}\, i \left\{ \kappa^{(2)} + a\frac{(1-\nu)e}{x^{(2)}} \right\} \sqrt{\frac{\alpha'}{2}\frac{4}{\varepsilon}}\sin(\pi\nu) , \\
\widetilde{g}(\nu; x^{(2)}; \overline{\kappa}^{(2)}; \overline{e}) &\equiv (x^{(2)})^{1-\nu}\, i \left\{ \overline{\kappa}^{(2)} - a\frac{(1-\nu)\overline{e}}{x^{(2)}} \right\} \sqrt{\frac{\alpha'}{2}\frac{4}{\varepsilon}}\sin(\pi\nu) .
\end{aligned} \tag{C.6}
$$

Plugging eqs. (C.3), (C.4) and (C.5) into eq. (3.16) and using the physical state conditions and the relation for the Laguerre polynomial

$$
\frac{d}{dx} L^{(\alpha)}_{\mathrm{n}}(x) = -L^{(\alpha+1)}_{\mathrm{n}-1}(x) , \tag{C.7}
$$

we obtain eq. (3.17).

D. Calculation of $\mathcal{A}_3$

In this appendix we give the steps to obtain eq. (4.10).

Introducing a Grassmann parameter η, we write the vertex operator $\mathbb{V}_A(\xi,\theta;k_M;\zeta_M)$ in an exponential form [25, 13]:

$$\begin{aligned}\mathbb{V}_A(\xi,\theta;k_M;\zeta_M) &= \int d\eta\ :\exp\left(i\sqrt{\frac{\alpha'}{2}}k_M\mathbb{X}^M+\frac{i}{2}\eta\zeta_M\dot{\mathbb{X}}^M\right)(\xi,\theta): \\ &= \int d\eta : \exp\left(i\sqrt{\frac{\alpha'}{2}}k_\mu\mathbb{X}^\mu+\frac{i}{2}\eta\zeta_\mu\dot{\mathbb{X}}^\mu\right):\times \\ &\quad\times:\exp\left[i\sqrt{\frac{\alpha'}{2}}\left(\kappa\mathbb{Z}+\overline{\kappa}\overline{\mathbb{Z}}\right)+\frac{i}{2}\eta\left(e\dot{\mathbb{Z}}+\overline{e}\dot{\overline{\mathbb{Z}}}\right)\right]: .\end{aligned} \tag{D.1}$$

In a similar way to eq. (C.2), we factorize the correlation function in the integrand on the r.h.s. in eq. (4.9) into two pieces; the piece depending on $\mathbb{X}^\mu$ $(\mu=0,\ldots,p)$ and that on $\mathbb{Z}$ and $\overline{\mathbb{Z}}$.

First we consider the former piece, namely the factor which depends on $\mathbb{X}^\mu$. We further divide this factor into the contribution from the bosonic coordinates and that from the worldsheet fermions. The bosonic contribution is the same as eq. (C.3) with the replacement $a\to\frac{\eta\theta}{\sqrt{2\alpha'}}$. The fermionic contribution becomes

$$\begin{aligned}&\langle 0|:\frac{1}{2}\zeta^{(1)}_\mu(\psi^\mu+\widetilde{\psi}^\mu)(-x^{(1)})::\exp\Big[-\theta^{(2)}\sqrt{\frac{\alpha'}{2}}k^{(2)}_\mu(\psi^\mu+\widetilde{\psi}^\mu)- \\ &\qquad\qquad -\frac{1}{2}\eta\zeta^{(2)}_\mu(\psi^\mu+\widetilde{\psi}^\mu)\Big](-x^{(2)}):\times \\ &\times\frac{1}{2}\zeta^{(2)}_\mu(\psi^\mu+\widetilde{\psi}^\mu)(-x^{(2)}):|0\rangle = \\ &= -\frac{\zeta^{(1)}\underset{(p)}{\cdot}\zeta^{(2)}}{x^{(1)}-x^{(2)}}+\eta\theta^{(2)}\sqrt{2\alpha'}\frac{(k^{(2)}\underset{(p)}{\cdot}\zeta^{(1)})(\zeta^{(2)}\underset{(p)}{\cdot}\zeta^{(3)})-(k^{(2)}\underset{(p)}{\cdot}\zeta^{(3)})(\zeta^{(1)}\underset{(p)}{\cdot}\zeta^{(2)})}{(x^{(1)}-x^{(2)})(x^{(2)}-x^{(3)})}\end{aligned} \tag{D.2}$$

Combining the results in the bosonic and the fermionic sectors and using the polarization conditions for $\zeta^{(a)}$ $(a=1,2,3)$, we find that the contribution from $\mathbb{X}^\mu$ to the correlation function in eq.(4.9) is

$$\begin{aligned}&\prod_{\mu=0}^{p}\delta(k^{(1)}_\mu+k^{(2)}_\mu+k^{(3)}_\mu)\prod_{1\le c<d\le 3}\exp\left[\frac{i}{2}\theta^{\mu\lambda}k^{(c)}_\mu k^{(d)}_\lambda\epsilon(\tau^{(c)}-\tau^{(d)})\right]\times \\ &\times\left(x^{(c)}-x^{(d)}\right)^{2\alpha' k^{(c)}\underset{(p)}{\cdot}k^{(d)}}\Big[-\frac{\zeta^{(3)}\underset{(p)}{\cdot}\zeta^{(1)}}{x^{(1)}-x^{(3)}}-\eta\theta^{(2)}\sqrt{2\alpha'}\times \\ &\qquad\times\frac{(\zeta^{(3)}\underset{(p)}{\cdot}\zeta^{(1)})\,G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)}+\overline{\kappa}^{(2)}e^{(2)})}{(x^{(1)}-x^{(3)})(x^{(1)}-x^{(2)})}+\eta\theta^{(2)}\sqrt{2\alpha'}\times\end{aligned}$$

$$\times \frac{(k^{(2)} \underset{(p)}{\cdot} \zeta^{(1)})(\zeta^{(2)} \underset{(p)}{\cdot} \zeta^{(3)}) - (k^{(2)} \underset{(p)}{\cdot} \zeta^{(3)})(\zeta^{(1)} \underset{(p)}{\cdot} \zeta^{(2)})}{(x^{(1)} - x^{(2)})(x^{(2)} - x^{(3)})} +$$

$$+ \eta\theta^{(2)}\sqrt{2\alpha'}\, \frac{(k^{(3)} \underset{(p)}{\cdot} \zeta^{(2)})(\zeta^{(3)} \underset{(p)}{\cdot} \zeta^{(1)})}{(x^{(1)} - x^{(2)})(x^{(2)} - x^{(3)})}\Bigg] . \tag{D.3}$$

Next we consider the remaining piece, namely the factor which depends on $\mathbb{Z}$ and $\overline{\mathbb{Z}}$, in the correlation function in eq. (4.9). Sending $x^{(1)} \to \infty$ and $x^{(3)} \to 0$, we find that this piece takes the form of

$$\begin{aligned}
&\langle 0|U_{\rm m}^{(-,-1)}(-x^{(1)}) : \exp\left[i\sqrt{\frac{\alpha'}{2}}\left(\kappa^{(2)}\mathbb{Z} + \overline{\kappa}^{(2)}\overline{\mathbb{Z}}\right) + \frac{i}{2}\eta\left(e^{(2)}\dot{\mathbb{Z}} + \overline{e}^{(2)}\dot{\overline{\mathbb{Z}}}\right)\right](x^{(2)},\theta^{(2)}) : \times \\
&\quad \times U_{\rm n}^{(+,-1)}(-x^{(3)})|0\rangle = \\
&= (x^{(1)})^{-(2{\rm m}+1)(1-\nu)}(x^{(2)})^{-2\alpha' G^{Z\overline{Z}}\kappa^{(2)}\overline{\kappa}^{(2)}} \exp\left[\mathcal{C}(\nu; k_m^{(2)}) + \eta\theta^{(2)}\sqrt{2\alpha'}\frac{G^{Z\overline{Z}}\overline{\kappa}^{(2)}e^{(2)}}{x^{(2)}}\right] \times \\
&\quad \times \langle \sigma, s|(\alpha_{1-\nu})^{\rm m} b_{\frac{1}{2}-\nu} \,\text{\textdiv}\, \exp\left[i\sqrt{\frac{\alpha'}{2}}\left(\kappa^{(2)}\mathbb{Z} + \overline{\kappa}^{(2)}\overline{\mathbb{Z}}\right) + \right. \\
&\qquad\qquad \left. + \frac{i}{2}\eta\left(e^{(2)}\dot{\mathbb{Z}} + \overline{e}^{(2)}\dot{\overline{\mathbb{Z}}}\right)\right](-x^{(2)},\theta^{(2)}) \,\text{\textdiv}\, (\overline{\alpha}_{-1+\nu})^{\rm n}\overline{b}_{-\frac{1}{2}+\nu}|\sigma, s\rangle,
\end{aligned} \tag{D.4}$$

with $x^{(1)} = \infty$ and $x^{(3)} = 0$. Here we have used the renormal ordering formula (2.11). The contribution from the bosonic coordinates Z and $\overline{Z}$ to the correlation function on the r.h.s. in eq. (D.4) has the form of eq. (C.5) with the parameter a replaced with $\frac{\eta\theta^{(2)}}{\sqrt{2\alpha'}}$. By using the commutation relation (2.8), we find that the contribution from the fermionic coordinates becomes

$$\begin{aligned}
&\langle s|b_{\frac{1}{2}-\nu} \,\text{\textdiv}\, \exp\left[-\left(\theta^{(2)}\sqrt{\frac{\alpha'}{2}}\kappa^{(2)} + \eta\frac{e^{(2)}}{2}\right)(\Psi + \widetilde{\Psi}) - \right. \\
&\qquad \left. - \left(\theta^{(2)}\sqrt{\frac{\alpha'}{2}}\overline{\kappa}^{(2)} + \eta\frac{\overline{e}^{(2)}}{2}\right)(\overline{\Psi} + \widetilde{\overline{\Psi}})\right] \,\text{\textdiv}\, \overline{b}_{-\frac{1}{2}+\nu}|s\rangle = \\
&= \frac{2}{\varepsilon}\left\{1 + \frac{\eta\theta^{(2)}}{\sqrt{2\alpha'}}\frac{2\alpha' G^{Z\overline{Z}}(\kappa^{(2)}\overline{e}^{(2)} - \overline{\kappa}^{(2)}e^{(2)})}{x^{(2)}}\right\} .
\end{aligned} \tag{D.5}$$

Gathering all the results obtained above and using the physical state conditions, we obtain the three point amplitude (4.10).

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